On lattice of Basic Z-Ideals

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Abstract

For an f-ring R with bounded inversion property, we show that $\mathrm{BZ(R)}$, the set of all basic z-ideals of R, partially ordered by inclusion is a bounded distributive lattice. Also, whenever R is a semiprimitive ring, $\mathrm{BZ}^0(\mathrm{R})$, the set of all basic z^0-ideals of R, partially ordered by inclusion is a bounded distributive lattice. Next, for an f-ring R with bounded inversion property, we prove that $\mathrm{BZ(R)}$ is a complemented lattice and R is a semiprimitive ring if and only if $\mathrm{BZ}^0(\mathrm{R})$ is a complemented lattice and R is a reduced ring if and only if the base elements for closed sets in the space $\mathrm{Max(R)}$ are open and R is semiprimitive if and only if the base elements for closed sets in the space $\mathrm{Min(R)}$ are open and R is reduced. As a result, whenever $\mathrm{R} = \mathrm{C(X)}$ (i.e., the ring of continuous functions), we have $\mathrm{BZ(C(X))}$ is a complemented lattice if and only if $\mathrm{BZ}^0(\mathrm{C(X)})$ is a complemented lattice if and only if X is a P-space.

Keywords: F-ring, lattice, Zariski topology, Semiprimitive ring, Reduced ring.

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