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Weak Armendariz Ideals

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Abstract

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In this article, we introduce the weak Armendariz ideals as a generalization of the Armendariz ideals and we examine its properties, its relation to other structures. Also by giving numerous examples and diverse, we evaluate the behavior of weak Armendariz ideals under some ring extensions.

Extended Abstract

Introduction

Throughout this article, R denotes an associative ring with unity. For a ring R, we denote by nil(R) the set of all nilpotent elements of R and by R[x] the polynomial ring with an indeterminate x over R. The symbol $T_n(R)$ stands for the ring of upper triangular matrices over a ring R. Recall that a ring R is called reduced if $a^2 = 0$ implies that a = 0, for all a ∈ R; R is semi-commutative if ab = 0 implies aRb = 0 for all a, b ∈ R; a ring R is said to be a NI ring if nil(R) forms an ideal. Liang introduced weakly semi-commutative rings as a generalization of IFP (i.e semi-commutative) rings. A ring is weakly semi-commutative ring if for any a, b ∈ R, ab = 0 implies arb is nilpotent element for any r ∈ R. Ghalandarzadeh et al. introduced the notion of an Armendariz ideal. They defined a left ideal of ring R to be an Armendariz if whenever polynomials $f(x)g(x) \in r_{R[x]}(I[x])$ where $g(x) = \sum_{j=0}^{n} b_j x^j$, $f(x) = \sum_{i=0}^{m} a_i x^i \in R[x]$ then $a_i b_j \in r_R(I)$ for all i, j. A one-sided ideal I of a ring R has the insertion-of factors property, or IFP if, for elements a, b of R the condition ab ∈ I implies aRb ⊆ I. semi-commutative property is called the insertion-of-factors property, or IFP. Observe that every completely semi-prime ideal of R has the IFP. As a generalization of annihilators, L. Ouyang and et al. introduced the concept of nilpotent annihilators. For a nonempty subset X of a ring R, they defined $N_R(X) = \{ a \in R \mid Xa \subseteq nil(R)\}$, which is called the nilpotent annihilator of X in R. In this paper, we introduce the notion of weak Armendariz and weak IFP ideals by considering the Nilpotent annihilators place of the right (left) annihilators. We show that the notion of weak Armendariz ideals generalizes that of Armendariz ideals introduced by Ghalandarzadeh. We study nilpotent annihilator Armendariz and nilpotent annihilator IFP property of certain subrings of matrix rings. Consequently, some known results are obtained as special cases and new families of weak Armendariz and weak IFP ideals are presented.

Results and discussion

We prove that the notion of weak Armendariz (one-sided) ideals generalizes that of Armendariz ideals of Ghalandarzadeh et al. Also, we will show all Armendariz ideals are NA-Armendariz, but there exists a NA-Armendariz ideal which is not Armendariz. Thus the NA-

Armendariz ideal is a true generalization of Armendariz ideal. Finally, we show that any left ideal of R is a weak Armendariz left ideal provided that R is Armendariz ring.

Conclusion

The following conclusions were drawn from this research.

- Let I be a weak Armendariz ideal of a ring R. Then, for any $n \in N$, $T_n(I)$ is weak Armendariz ideal.
- For every Armendariz left ideal $I \neq 0$ of R, $T_n(R)$ is a weal Armendariz ideal which is not Armendariz ideal.
- Suppose that I is a one-sided ideal of a ring R and Δ is a multiplicatively closed subset of R consisting of central regular elements. If I has the N-FIP in R, then $\Delta^{-1} I$ has the N-FIP in $\Delta^{-1}R$.

Keywords: Right annihilator, Nilpotent annihilator, Weak Armendariz ring, Semi-commutative ring.

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