The Numerical Solution of Fractional Parabolic Partial Integro-Differential Eequations by Gaussian and Inverse Multiquadric Radial Basis Functions

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Extended Abstract

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Introduction

Many mathematical formulations of physical phenomena contain integro-differential equations. These equations arise in fluid dynamics, biological models, chemical kinetics, ecology, control theory of financial mathematics, aerospace systems, industrial mathematics etc. It is worth mentioning that integro-differential equations are usually difficult to solve analytically, and so it is required to obtain an efficient approximate solution for them.

Fractional calculus deals with derivatives and integrals of arbitrary real or complex orders. This subject has attracted attention of many scientists in mathematics, physics and engineering. So, it has become a hot issue in recent years.

Fractional integro-differential equations arise in the mathematical modelling of various physical phenomena, such as heat conduction in materials with memory. Moreover, these equations are encountered in combined conduction, convection and radiation problems. There are only a few techniques for the solution of fractional integro-differential equations, since it is relatively a new subject in mathematics. Some of these methods are Legendre spectral tau method, Adomian decomposition method, piecewise polynomial collocation methods, spline collocation method, hybrid collocation method, hybrid functions approximation by block-pulse functions and Bernoulli polynomials, Taylor expansion approach, differential transform method and wavelet methods.

In recent years many problems in mathematics, physics and engineering have been numerically solved by radial basis functions (RBFs) methods. In this paper, we focus on the Gaussian and inverse multiquardic RBFs as two of the most important tools in engineering and sciences to solve a class of fractional parabolic integro-differential equations. This class of equations describes some phenomena in compression of viscoelastic media and nuclear reactor dynamics.

Material and methods

In the proposed method, first the fractional derivative operator is transformed into a non-singular equivalent. Then, the Gaussian and inverse multiquardic RBFs together with the collocation method and Gauss-Legendre quadrature formula are used to transform the problem under consideration into the corresponding system of linear algebraic equations, which can be simply solved to achieve an approximate solution of the problem.

Results and discussion

Some numerical examples are examined to demonstrate the efficiency and high accuracy of the present method. The obtained results demonstrate that there is a good agreement between the approximate solutions and the exact ones. Also we hope that the proposed method can provide numerical solutions with high accuracy for the problems under study for all fractional orders. Meanwhile, the best value for the shape parameter in the Gaussian and inverse multiquardic RBFs method can be obtained by employing an appropriate optimization method.

Conclusion

The following conclusions were extracted from this research.

- The established method transforms such problems into equivalent systems of algebraic equations by expanding the solution of the problem in terms of the RBFs and applying Gauss-Legendre integration formula.
- Only a few number of the RBFs is needed to obtain a high accurate numerical solution for such problems.
- The presented method can easily be developed for other classes of fractional partial integro-differential equations.

Keywords: Fractional parabolic partial integro-differential equations, Gaussian radial basis functions (RBFs), Inverse multiquardic RBFs, Collocation method, Gauss-Legendre quadrature formula.

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