Results in Injective Envelope and Indecomposable Injective Modules

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Extended Abstract

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Introduction

Throughout this paper, R is a commutative ring with non-zero identity and M is an R-module. The study of injective modules is very important in commutative algebra and homological Algebra. Any product of (even finitely many) injective modules is injective; conversly, if a direct product of modules is injective, then each module is injective. Every direct sum of finitely many injective modules is injective. In general, submodules, factor modules, or infinite direct sum of injective modules need not be injective. Every submodule of every injective module is injective if and only if the ring is Artinian semisimple. Also every factor module of every injective module is injective if and only if the ring is Hereditary. Finally every infinite direct sum of injective modules is injective if and only if the ring is Noetherian. In this paper we study some new propertis of this modules.

Material and methods

The main tool used in the proofs of the main results of this paper is the properties of injective modules and injective envelopes.

Results and discussion

We present some new properties of injective envelopes, injective modules, prime submodules and maximal submodules.

Conclusion

We prove the following results:

1. Over finitely generated multiplication modules, every prime submodule is irreducible.
2. If N is a prime submodule of finitely generated multiplication R-module M such that E(M/N) is finitely generated, then N is a maximal submodule of M. Also we give several corollaries for this note.
3. Also we find relations beetwen Artinian ring, Noetherian ring, indecomposable injective modules and injective cogenerators of module**s.**

Keywords: Associated prime ideals, Injective envelope, Injective modules, Artinian modules, Noetherian modules.

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