Operator Monotone Functions and Convexity of Its Derivatives Norms

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Extended Abstract

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Introduction

Given the important role convex and quasi-convex functions play in many areas of mathematics and especially in optimization, one of the inequalities that has attracted the attention of many mathematicians in recent decades is Hermit-Hadamard's famous inequality. Significant generalizations and refinements have been obtained for this inequality in a diverse variety of convexity, including convex operator functions of self adjoint operators on Hilbert spaces, matrix functions, quasi-convex, s-convex and log-convex functions.
In this paper, we generalize this inequality to differentiable functions whose norm of their derivatives are convex functions.

Results and discussion

In this paper, we consider differentiable mappings which norm of the induced maps by them on the set of self adjoint operators is convex, quasi convex or s-convex. We show that if f is an operator monotone function on $(0,\infty)$, A is a strictly positive operator and $|||.|||$ a unitarily invariant norm, then $|||D^n f(A)||| \leq ||f^{(n)}(A)||$for all positive integers n. We also prove that $||f^{(n)}(.)||$ is a quasi-convex function on the set of all strictly positive operators in B(H). Examples and applications for particular cases of interest are also illustrated. Finally, an error estimate for the Simpson formula is addressed.

Conclusion

The following conclusions were drawn from this research.

- As an important application of the results in this paper, we find bounds for $||f(B)-f(A)||$ in terms of $||B-A||$, which is one of the central problems in perturbation theory.
- We establish some estimates of the right hand side of some Hermite-Hadamard type inequalities in which differentiable functions are involved, and norms of the maps induced by them on the set of self adjoint operators are convex, quasi-convex or s- convex.

Keywords: Hermite-Hadamard inequality, Differentiable functions, Unitarily invariant norms, Operator monotone functions.

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