Gray Images of Constacyclic Codes Over Some Polynomial Residue Rings

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Extended Abstract

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Introduction

The problem of linking codes over a ring R to codes over a finite field F_q, $q = p^m$, via the so called Gray maps, has been of much interest in recent years. Also, among the rings, many kinds of finite chain rings, and among the codes, the class of constacyclic codes have been of more interest. In all of the researches in this topic, a ring R is considered and a special kind of a Gray map, say ϕ, over R is introduced. Then it is verified that ϕ has a special ability that caries a class of constacyclic codes over R to a class of constacyclic codes over F_q.

We take another look to the above problem. Let us choose the ring R, a class of constacyclic codes over R, say λ-constacyclic codes, and a class of constacyclic codes over the field F_q, say θ-constacyclic codes. Now let us ask for existence and properties of all Gray maps ϕ over R that caries all λ-constacyclic codes over R to θ-constacyclic codes over F_q. We call such a map, a (λ, θ)-Gray map over R. Answering to this question leads to a more general results in this topic. In our work, we consider the polynomial residue ring $R_s = \frac{F_{p^m}[u]}{\langle u^s \rangle}$ and will characterize all (λ, θ)-Gray maps over R_s.

Method

Our strategy to classify all (λ, θ)-Gray maps over R_s is to obtain a more practical equivalent condition for a map ϕ to be a (λ, θ)-Gray map. In fact, we obtain a system of equations that solving this system is equivalent to finding ϕ. Then we try to find conditions that guarantee the existence of a solution to the system.

Results

The following results were obtained in this paper.

- If ϕ is a (λ, θ)-Gray map of length l over R_s and $\lambda = \lambda_0 + \lambda_1 u + \cdots + \lambda_{s-1} u^{s-1}$ then we must have $\theta = \lambda_0^{\,l}$.
- The characteristic of F_q must divide the length of the Gray map; that is $p|l$. This implies that the Gray images of all constacyclic codes over R_s are repeated-root constacyclic codes over F_q.
- If $\theta = 1$ then we must have $\lambda_0 = 1$.
- All (λ, θ)-Gray maps over R_2 were classified.
- A (λ, θ)-Gray map of length p^k over R_{p^k} was introduced.

Keywords: Gray map, Constacyclic code, Chain ring, Polynomial residue ring.

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