Strong Well-Posedness for a Class of Split Variational Inequalities with Set-Valued Maps

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Extended Abstract

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Introduction

The concept of well-posedness, which was firstly introduced by Tykhonov in 1966 for a minimization problem, has been studied widely in recent years, for equilibrium problems, variational inequalities, optimization problems, fixed point problems, etc. It requires the existence and uniqueness of its solution and the convergence to the unique solution of its approximating sequences. There are a great many kinds of generalizations for the concept of well-posedness, such as parametric well-posedness, Levitin-Polyak well-posedness, and α-well-posedness. Due to the close correlation between variational inequalities and optimization problems, Lucchetti and Patrone in 1981 generalized the concept of well-posedness to the variational inequalities. Split variational inequality consists of two variational inequalities with a linking linear constraint, which was introduced by Censor et al. in 2012 can be regarded as a generalization of variational inequalities. Thus, the concepts of well-posedness can be generalized to the split variational inequalities. In this paper, we present a generalization of well-posedness for a system of split multi-valued variational inequalities with set-valued maps and establish a metric characterization for them. Moreover, we show that the strong well-posedness is equivalent to the existence and uniqueness of solution for a split multi-valued variational inequality.

Material and methods

In this scheme, first we present split multi-valued variational inequalities in terms of the normal subdifferential for set-valued maps and give a new formulation for them. Then, by using the concept of diameter for approximate solution sets, we obtain a characterization for well-posedness of split multi-valued variational inequalities.

Results and discussion

By using the normal subdifferential, we establish some equivalent results for the strong well-posedness of split multi-valued variational inequalities. Also, we obtain some metric characterizations for the strong well-posedness of split variational inequalities.

Conclusion

The following conclusions were drawn from this research.

- The concept of well-posedness is generalized to a system of split variational inequalities with set-valued maps. Also, Normal subdifferential in the sense of Mordukhovich is used for set-valued maps, which is not necessarily convex.
- Some metric characterizations of well-posedness for a class of split variational inequalities are established.

Keywords: Approximating sequence; Well-posedness; Variational inequality; Normal subdifferential.

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