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A Numerical Simulation of Anomalous Electro-Diffusion of Ions in Spiny Dendrites Using a Local Petrov-Galerkin Method

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Extended Abstract

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Introduction

During two recent decades, the theory of fractional calculus has gained notable attention from scientists due to its capability in modeling and describing many complicated phenomena in physics, engineering sciences, medicine, biologic, economy and etc. Indeed many of phenomena, especially anomalous processes, could be successfully described by using the theory of fractional calculus. Recently, the theory of fractional calculus is used to extend the modeling capabilities of many mathematical models in bioengineering. The cable equation is one of the most fundamental mathematical models in the neuroscience. Indeed the cable equation is derived from the Nernst–Planck equation which has been used in modeling the electrodiffusion of ions in smooth homogeneous cylinders. Nevertheless, the standard Nernst–Planck equation and its simplification, the classical cable equation, cannot exactly describe the anomalous diffusion processes in biological systems (e.g., anomalous electrodiffusion of ions in spiny dendrites). Recently, the fractional forms of the Nernst–Planck and cable equations for describing anomalous electrodiffusion of ions in spiny dendrites have been introduced, and it has been shown that the new fractional models are better than their traditional forms. Since finding an analytical solution for the fractional problem is a very difficult or often impossible task, various computational techniques have been introduced and developed for investigating these problems.

In recent years, an advanced class of numerical schemes called meshless methods have been widely and successfully used to approximate the solution of several types of mathematical and engineering problems. These methods can be applied for complicated, irregular and high dimensional spatial domains because these processes don’t require to mesh the domain such as mesh-based methods and instead of them, a set of scattered data points on the domain is considered. Meshless methods based on the radial basis functions(RBFs) are one of the most efficient and powerful class of computational techniques. The RBFs meshless methods are divided into two main classes: RBFs meshless methods based on strong form such as radial basis functions collocation methods and RBFs meshless methods based on weak form such as local radial point interpolation method.

The main goal of this work is to propose a meshless computational strategy based on the local weak form for the numerical solution of nonlinear time fractional cable equation.

Material and methods

In this study, firstly, the nonlinear time fractional cable equation is discretized in the time direction by implementing an implicit time stepping procedure with second-order accuracy. It has been proved that the proposed time discretization scheme is unconditionally stable. Then, a meshless method based on a combination of the local Petrov–Galerkin weak form and a collocation approach is implemented for spatial discretization. The presented combination technique leads to a nonlinear system of algebraic equations. In our implementation to avoid solving a nonlinear system and for obtaining acceptable results, a predictor–corrector algorithm is used.

Results and discussion

The performance and accuracy of the proposed technique to deal with the problem are investigated through the three test problems with known exact solutions. In our computational process, a set of uniformly distributed nodes is considered as data sites and evaluation points. The spatial shape functions are generated on the distributed field nodes by using the radial point interpolation method, in which GMQ and TPS functions are used as the radial basis function.
The reported results through the figures and tables show the accuracy and efficiency of the presented method. Also, the results reveal that the process is very appropriate for solving nonlinear two-dimensional time fractional cable equations.

Conclusion

The following conclusions were drawn from this research.

- The results show the second-order accuracy and stability of the proposed time discretization scheme.
- The proposed meshless method based on the local weak formulation of the governing problem is an efficient and accurate technique to deal with complicated natural problems.
- The data-dependent radial point interpolation shape functions are easily constructed. These shape functions possess the delta Kronecker property and can be easily used to problems with essential boundary conditions. Moreover, they are powerful tools to approximate the multi-variable functions.
- The predictor–corrector method is an efficient approach to deal with nonlinear problems.

Keywords: Nonlinear Cable equation, Fractional differential equation, Radial point interpolation method, Meshless local Petrov–Galerkin, Stability analysis

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