On the Lattice Structure of the Space of all Bochner Integrable Banach Lattice-Valued Functions

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Extended Abstract

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Introduction

   Suppose $(X, \sum, \mu)$ is a finite measure space, E is a Banach lattice, and $B(X, E, \mu)$ is the space of all Bochner integrable E -valued functions.

   In this note, we show that $B(X, E, \mu)$ is a KB-space or has the sequential Fatou property if and only if so is E. Among this, some results about Bochner integral convergence in $B(X, E, \mu)$, using order structure of E, have been proved, as well.

   Let us start with some motivation. Bochner integral is one of the best ways to generalize the usual notion of integral (Riemann integral or Lebesgue one) to vector-valued functions. So, it is an enlightening line for research to discover different aspects of the space of all Bochner integrable functions. Certainly, investigative properties depend on the structure of the underlying vector space; for example, we can not expect order structure in the space of all Bochner integrable E -valued functions when E is a Banach space, in general.

 Let $(X, \sum, \mu)$ be a finite measure space and E be a Banach lattice. The space of all Bochner integrable E -valued functions from X into E is denoted by $B(X, E, \mu)$. It is shown that it is a Banach lattice under norm $||f|| := \int ||f(w)|| d\mu(w)$.

   Let us again consider some motivation. Suppose that E is a Banach lattice. It is known that in some classical function spaces such as l_p for $1 \le p \le \infty$ or c_0, unbounded order convergence (uo-convergence, in brief) for nets is as the same as pointwise convergence; in addition, in $L^p(\mu)$-spaces, uo -convergence for sequences and almost everywhere convergence agree. So, uo -convergence is a generalization of coordinate-wise convergence in general Banach lattices; furthermore, for order bounded nets, uo -convergence and order one coincide. So, it is of independent interest to investigate some theorems such as the monotone convergence theorem which relies on the pointwise convergence, by means of order convergence. Beside this, some attempts have been made to generalize such theorems to monotone Banach lattice-valued functions using pointwise convergence .

   In this paper, we try to generalize this result using unbounded order and order convergences.

Results

   In this paper, we investigate whether $B(X, E, \mu)$ is a KB -space or is sequentially Fatou. More precisely, we formally prove this holds exactly when E possesses the same property.

Conclusions

- $B(X, E, \mu)$ is a $KB - space$ if and $only$ if so is E.
- $B(X, E, \mu)$ possesses the sequential Fatou property if and only if so is E.
- Suppose $B(X, E, \mu)$ is a finite measure space and E is a σ −order continuous Banach lattice. In addition, assume that (f_n) is an almost order bounded sequence of Bochner

integrable functions from X into E such that $f_n \xrightarrow{uo} f$. Then, f is Bochner integrable and $\int f_n d\mu \to \int f \, d\mu.$

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