Weak Multilevel Path Simulation for Jump-Diffusion Assets

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Extended Abstract

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Introduction

In many financial engineering applications, we are interested in the expected value of a financial derivative whose payoff depends on the solution of a stochastic differential equation (SDE). Introduced by Giles in 2008, the multilevel Monte-Carlo (MLMC) approach reduces the computational complexity of such expectations using the standard Monte-Carlo method. The efficiency of the MLMC method is influenced by the strong convergence order of the discretization. Until 2014, MLMC was just applicable with strong numerical schemes. The first attempt to remove this dependency was due to Giles and Szpruch when they introduced antithetic MLMC. Belomestny and Nagapetyan, in 2017, as the second attempt, improved the MLMC approach to be applied with weak approximation methods as well. It was called the weak MLMC scheme and they proved the computational cost with the weak Euler method is the same as the classical MLMC with the strong Euler method.

Inspired by recent advances in the application of the multilevel Monte-Carlo (MLMC) approach to Lévy driven assets, we benefit from this method to price financial derivatives. Note the solution of stochastic differential equations as a powerful tool to model the dynamics of asset prices are of much importance. Also, the no-arbitrage pricing theory suggests the price of a financial derivative like an option as the expectation of the payoff discounted by an appropriate time factor.

In this paper, as an improvement of Belomestny's work and with a new approach in the theory, we express and prove the convergence theorems in $L^p(\Omega)$ space for $p \geq 2$ and not only 2. We also seek to implement the weak MLMC algorithm for nonlinear equations with dependent components X_i and X_j.

Material and methods

Using the weak Euler method, we calculate the numerical estimate of the underlying asset, which satisfies a multi-dimensional stochastic differential equation with Lévy noise, and then applying the weak multilevel Monte-Carlo method the expected price is obtained. For the new theoretical point of view, we need some requirements which are stated and proved as lemmas and theorems throughout the paper.

Results and discussion

We show some numerical experiments when applied to different types of processes. We price European call options both for diffusion and jump-diffusion asset prices using weak MLMC algorithm. We also test the method with the Heston model asset price which doesn't

satisfy the Lipchitz requirement for the underlying SDE. Surprisingly, the results are acceptable in all cases.

Conclusion

In this paper, we price the options in multidimensional space using the weak Euler method together with the MLMC algorithm. It is observed that for the Euler method, weak convergence and reduction of variance are of the first order and the computational cost is reduced compared to the Monte-Carlo method. We proved the convergences in $L^p(\Omega)$ space for $p \geq 2$ and not just 2. Numerical examples show the result of the implementation of the algorithm for call options with nonlinear equations and dependent components.

Keywords: Numerical solution of stochastic differential equations (SDE), weak multilevel Monte-Carlo method (MLMC), weak approximations, Euler scheme, Lévy process, Jump-diffusion process.

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