# Solving Special Case of Inverse Sturm-Liouville Problem with Aftereffect by using Chebyshev Polynomials

Shahrbanoo Akbarpoor[1], Abdol Hadi Dabbaghian[*2], Abdolali Nematy[3]
1. Department of Mathematics, Jouybar Branch, Islamic Azad University, Jouybar, Iran
2. Department of Mathematics, Neka Branch, Islamic Azad University, Neka, Iran
3. Department of Mathematics, University of Mazandaran, Babolsar, Iran

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## Extended Abstract

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### Introduction

In this study, we consider the differential equation with aftereffect under the separated boundary conditions on a finite interval. In fact, we consider the Sturm-Liouville operator disorganized by a Volterra integral operator. We obtain the numerical solution for the special case of the inverse aftereffect problem by applying Chebyshev interpolation method by calculating the solution of the integro-differential equations.

In section 2, we show the asymptotic form of the solution and the eigenvalues of the problem and present the uniqueness theorem for the solution of the inverse aftereffect problem. In section 3, we approximate the function M in the special case contained $q(x) \equiv 0$, by using the method of Chebyshev interpolation and provide the numerical algorithm for solving the inverse aftereffect problem.

### Preliminaries

In this section, our goal is to show asymptotic form of the solution and the eigenvalues of the problem and to present the uniqueness theorem for the inverse aftereffect problem under the given boundary conditions.

### Numerical algorithm

In this section, we describe a numerical method based on Chebyshev interpolation method by using Chebyshev polynomials of the first kind for solving the inverse aftereffect problem by calculating the solution of the integro-differential equations.

Since the solution of the integral equation is the solution of inverse problem, so it is sufficient that we solve the integral equation. We apply Chebyshev interpolation method for solving the integral equation. We use Chebyshev polynomials of the first kind as the basic functions for calculating the approximation of the function M and convert the integral equation to the system of the linear equations. We apply Matlab software program for drawing the figures.

### Conclusion

In this study, we applied Chebyshev polynomials of the first kind to get the approximation of the solution of inverse problem for the special case of the aftereffect equation under the separated boundary conditions. Also, we provided some examples to calculate the numerical values of the function M and showed the stable numerical results in the presented examples.

**Keywords:** Inverse problem, aftereffect equation, Chebyshev polynomials.

[*]Corresponding author: a.dabbaghian@umz.ac.ir