# Some Applications of Casorati Curvature for Statistical Submanifolds of Sasakian Statistical Manifolds and Locally Homogeneous, Quasi-Umbilical Hypersurfaces

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## Extended Abstract

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**Introduction**

The main framework in this paper is affine geometry. Affine geometry has many applications in mathematics and other scientific fields. The geometry of statistical manifolds based on information geometry, affine differential geometry and Hessian geometry. At first, we have a spacious explanation of necessary introduction in rather different subjects. In this part, statistical submanifolds of Sasakian statistical manifolds with constant φ-sectional curvature is considered as the pivotal topic. Afterwards, with a rather long process, we obtain an optimal inequalities between generalized normalized scalar curvature as an intrinsic property and δ-Casorati curvature as an extrinsic property. We also prove that the equality holds if and only if all corresponding components of second fundamental forms of doual connections for a statistical manifold only have different sign. This results are prove for generalized normalized δ(r;m)-Casorati curvatures in two cases, $0 < r < m(m + 1)$ and $r > m(m + 1)$, where m is the dimension of manifold. We know that Casorati curvature have more capability than sectional curvature . Therefore, in the second part of paper, we deduce some results about locally symmetric, quasi-umbilical hypersurfaces of real space forms with zero curvature, using Casorati curvature. In more details, we derive some equations which our manifold is affine equivariant with a convex part of one of hypersurface with these equations. Thus, we reach to some computational classifications.

**Material and methods**

In first part, we use the properties of tensor fields correspond to statistical structure, Gauss equations, definition of Casorati curvature and definition of φ-constant curvature for prove the main theorem. The equality of scalar curvature and Casorati curvature is the basic tool in the proofs of theorems in the second part.

**Results and discussion**

Existence of an inequality between normalized scalar curvature and Casorati curvature which related an intrinsic property to an extrinsic property of a statistical submanifold of a Sasakian statistical manifold with φ-constant curvature.
Analytical and algebraic expressions for locally symmetric, quasi-umbilical hypersurfaces that concludes the usability of affine geometry in work with softwares.

**Conclusion**

The main conclusions of this paper are as follows:

- The following inequality between normalized scalar curvature and Casorati curvature

$$\rho \leq \frac{1}{m(m+1)}\bar{\delta}_{\mathcal{C}}(r;m) + \frac{1}{m}\hat{\mathcal{C}} - \frac{2(m+1)}{m}\|\hat{H}\|^2 + \frac{(m+1)}{m}\tilde{g}(H.H^*)$$

$$+\frac{3(c-1)}{4m(m+1)}\|P\|^2 \quad + \frac{1}{4(m+1)}[c(m-1) \; + 3m \; + 5].$$

- Locally homogeneous isometrically immersed hypersurfaces of $\mathbb{R}^{m+1}$ with equal normalized scalar curvature and $\delta$-Casorati curvature can be affine equivariant with a convex part of the following hypersurface :

$$(y - \tfrac{x^2}{2} - \tfrac{1}{2}\textstyle\sum_{i=1}^{m-2}\tfrac{u_i^2}{z})^{m+1} z^m = 1.$$

- In cases $m = 3,\ 4, 5$ we have another classifications for locally homogeneous isometrically immersed hypersurfaces of $\mathbb{R}^{m+1}$ with equal normalized scalar curvature and $\delta$-Casorati curvature.

**Keywords:** Casorati curvature, Local homogeneous**,** Quasi-umbilical, Sasakian manifolds, Statistical manifolds.

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