Existence of Solutions of Infinite Systems of Non-Linear Ontegral Equations of Volterra Type in Fréchet Space $C(\mathbb{R}_+^N, \mathbb{R}^\infty)$

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Extended Abstract
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Introduction

Measures of noncompactness are useful tools in the theory of operator equations in Banach spaces and Fréchet spaces. They are frequently used in the theory of functional equations, including differential equations with partial derivatives, integral equations, integro-differential equations, optimal control theory. The measures of noncompactness and Darbo fixed point theorem play important roles in fixed point theory and its applications. The first measure of noncompactness was defined and studied by Kuratowski in 1930. In 1955 Darbo, using the concept of a measure of noncompactness, proved a theorem guaranteeing the existence of fixed points of the condensing operators. The theory of infinite systems of integral or differential equations are an important branch of nonlinear analysis. The measure of noncompactness has been applied for existence of solution of non-linear integral equations, ordinary differential equations and finite and infinite systems of differential equations by different researchers. Also, they have many applications in our real world problems such as mechanical engineering, the theory of branching processes and the theory of neural networks. Suppose that $\mathbb{R}^\infty$ is the linear space of all real sequences. Recently, Olszowy has described a new Fréchet space as $C(\mathbb{R}_+, \mathbb{R}^\infty)$ (the space of all continuous functions defined on $\mathbb{R}_+ = [0, \infty)$ with values in $\mathbb{R}^\infty$) and has investigated some of its properties and has defined a measure of noncompactness on this space. Then using Darbo's fixed point theorem, she has dealt with the solvability of an infinite system of integral equations. Also Latifa et al. have investigated the solvability of a functional integral equation in the Fréchet space $C(\Omega)$ ($\Omega \subseteq \mathbb{R}^N$) and by defining a family of measures of noncompactness, they have solved a system of integral equations in this space.

Material and methods

In this scheme, first we define a measure of noncompactness on the Fréchet space $C(\mathbb{R}_+^N, \mathbb{R}^\infty)$ and we establish an abstract fixed point theorem of Darbo type in this space. Then by using it we investigate the existence of solution for a nonlinear integral equation of Volterra type.

Results and discussion

We investigate the existence of solution for a nonlinear integral equation and then solve two illustrative examples to verify the effectiveness and applicability of our results.

Conclusion

The following conclusions were drawn from this research.

- We define a measure of noncompactness on the Fréchet space $C(\mathbb{R}_+^N, \mathbb{R}^\infty)$.
- We establish an abstract fixed point theorem of Darbo type in the space $C(\mathbb{R}_+^N, \mathbb{R}^\infty)$.
- We investigate the existence of solution for a nonlinear integral equation of Volterra type.

Keywords: Measure of noncompactness, Fréchet space, Darbo's fixed point theorem, Infinite system of integral equations of Volterra type.

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