Finite Element Method for Solving Nonlinear Inverse Diffusion Problem

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Extended Abstract

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Introduction

There are important applications of inverse problem in geophysics, electromagnetics, scattering, and various detection techniques. Also, this concept has used widespread acceptance in applied mathematics. In general, this problem is often ill-posed, in other hand, a small error in one's measurement may lead a big variation in the model determination, and large scale in computation. Its solution does not satisfy the general requirement of existence, uniqueness and stability under small changes to the input data. To overcome such difficulties, a variety of techniques for solving this problem has been proposed, where have been resulted from mathematical fields such as partial differential equations, numerical analysis, harmonic analysis, functional analysis, Fourier analysis and etc. Among the most versatile methods the following can be mentioned: Tikhonov regularization, iterative regularization, base function method (BFM) and the function specification method (FSM).

In the case of inverse diffusion problem, we have an inverse problem for a parabolic partial differential equation (PDE), where the solution of this problem requires to determine an unknown diffusion coefficient from an additional information. This new data is usually given by adding small random errors to the exact values from the solution to the direct problem. Various methods have been developed for the analysis of inverse diffusion problems involving the estimation of diffusion coefficient from measured data inside the material.

The finite element method (FEM) is known as very powerful tool for solving differential equation which was first introduced by Courant in 1943. This method is applied for solving linear and nonlinear problems by many researchers. Milos Zlamas used FEMs for nonlinear parabolic equations. El-Azab and Abdelgaber obtained finite element solution of nonlinear diffusion problems. Volker John and Ellen Schmeyer applied this method for time-dependent convection-diffusion-reaction equations with small diffusion. Wolfgang Bangerth applied adaptive FEMs for nonlinear inverse problems. Larisa Beilina and Johnson used a hybrid finite element/difference method for an inverse scattering problem. Larisa Beilina applied adaptive finite element/difference method for inverse elastic scattering waves. Xianwu Ling used a non-iterative FEM for inverse heat conduction problems.

In this paper, a numerical method based on the finite element method and the least square scheme with the Tikhonov regularization method for nonlinear inverse diffusion problem is presented. For this propose, first finite element method and basis functions will be used to discretize the variational form of the problem; then the least square scheme and Tikhonov regularization method are proposed to correct diffusion. It is assumed that no prior information is available on the functional form of the unknown diffusion coefficient in the present study, and so, it is classified as the function estimation in inverse calculation. Numerical result shows that a good estimation on the unknown functions of the inverse problem can be obtained.

Material and methods

The main purpose of this study is to construct a numerical technique for nonlinear inverse diffusion problem by using lagrange polynomials with FEM, least square scheme and Tikhonov regularization method to obtain approximate solution and diffusion coefficient at any time step based on measurements data. Also, stability and error estimation, considering some conditions on the problem, are proposed.

Results and discussion

The present study, successfully applies the numerical method involving the FEM with the least square scheme and the Tikhonov regularization method to a nonlinear in verse diffusion problem. From the illustrated example it can be seen that the proposed numerical method is efficient and accurate to estimate the thermal diffusivity in a one dimensional nonlinear inverse diffusion problem. Numerical result show that an excellent estimation can be obtained within a couple of minutes CPU time at Core (i5)–2.67 GHz PC. We also apply other different sets of the initial guesses, such as $\{a_{۰}, a_{۱}, ..., a_q\} = \{۰.۲, ۰.۲, ..., ۰.۲\}$, $\{۰.۷, ۰.۷, ..., ۰.۷\}$ and $\{۱.۱, ۱.۱, ..., ۱.۱\}$, results show that the effect of the initial guesses on the accuracy of the estimates is not significant for the present method.

Keywords: Nonlinear inverse diffusion problem, Finite element method, Least square method, Tikhonov regularization method, Error estimation.

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